

Logic and Inference Rules

Introduction

Knowledge representation

- _ Has been studied long before the emerge of the WWW in the field of artificial intelligence, or philosophy(Aristotle, the father of logic)
- _ Logic is the foundation of knowledge representation

Why is logic important for us?

- _ It provides high-level language
- _ It has well understood formal semantics
- _ There exists precise notion of logical consequence
- _ There exists proof systems which make possible to trace the proof that leads to a logical consequence
- _ It provides explanations for answers

Predicate logic

Can be divided in subsets:

Description logic

- Corresponds to OWL Lite and OWL DL

Rules systems (Horn logic, definite logic programs)

Rules

A rule has the form: $A_1, A_2, \dots, A_n \rightarrow B$

where A_1, \dots, A_n, B atomic formulas

There are two interpretations of this rule:

- If A_1, \dots, A_n are known to be true, then B is also true
- If the conditions A_1, \dots, A_n are true, then carry out the action B

Observation1

Description logics and Horn logic are orthogonal

Example

A person studies and lives in the same city.

We want:

- to say that this person is “home student”
- to say that the person is either man or woman

Rules Clasification(1)

There are different types of rules

Example

A vendor who wants to give special discount if it is customer 's birthday

R1:If birthday, then special discount

R2:If not birthday, no special discount

The system works when the provided information is complete

Rules Clasification (2)

What if customer 's birthday is unknown?

R1:If birthday, then special discount

R2:If birthday unknown, no special discount

This system of rules is applied when information is incomplete

Rules Clasification(3)

- _ MONOTONIC Rules

- remain valid even when new knowledge becomes available

- _ NONMONOTONIC Rules

- becomes invalid when new information is added

Monotonic Rules(1)

Example: Family relations

Base predicates:

Mother(X,Y)-X is mother for Y

Male(X)-X is male

Derived predicates:

Male(X),Parent(P,X),Parent(P,Y),Notsame(X,Y)
→Brother(X,Y)

Monotonic Rules(2)

Syntax:

$\text{loyalCustomer}(X), \text{age}(X) > 60 \rightarrow \text{discount}(X)$

Variables: X

Constants: 60

Predicates: loyalCustomer

Function symbols : age

Monotonic Rules(3)

Rule: $A_1, A_2, \dots, A_n \rightarrow B$

We call:

B-head of the rule

$\{A_1, \dots, A_n\}$ –body of the rule

A_1, A_2, \dots -devices

Monotonic Rules(4)

Facts

LoyalCustomer(b23458) says that the customer with the given id is loyal.

Logic Programs

A program is a finite set of facts and rules

Goals

A goal denotes a query asked to a logic program

Nonmonotonic Rules(1)

In the nonmonotonic rule system, one rule may
- not be applied, even if all the premises are proven

Example

$p(X) \rightarrow q(X)$

$r(X) \rightarrow \text{non}(q(X))$

CONFLICT!!!!!!

Nonmonotonic Rules(2)

The conflict can be resolved using priorities among rules

We can use the following notation to specify the fact that the rule r_1 is stronger than r_2 .

$r_1: p(X) \rightarrow q(X)$

$r_2: r(X) \rightarrow \text{non}(q(X))$

$r_1 > r_2$

Nonmonotonic Rules(3)

Note:

It is required that the priority relation to be acyclic

Syntax

$r : L_1, L_2, \dots, L_n \rightarrow L$

L –the head of the rule

$\{L_1, \dots, L_n\}$ -the body of the rule

L_1, L_2, \dots -literals

Nonmonotonic Rules(4)

Defeasible Logic Program

Is a triple $(F,R,>)$ consisting of:

- a set of facts F
- a finite set of defeasible rules
- an acyclic binary relation

Nonmonotonic Rules(5)

An example

Imagine that we want to rent an apartment,
which should:

- _ be at least 45sq m
- _ have at least 2 bedrooms
- _ if it on the 3rd floor or higher, there must be a lift
- _ pets allowed

rent offered :300 euro in the city centre ,
250 euro suburbs

You are willing to pay 5 euro/sq m for a larger
apartment and 2 euro/sq m of garden

The total amount shouldn't be bigger than 400

Priorities:

- the price
- the presence of garden
- additional space

Formalisation of requirements

- $\text{size}(x,y)$ – y is the size of apartment x
- $\text{bedrooms}(x,y)$ – x has y bedrooms
- $\text{price}(x,y)$ - ...
- $\text{floor}(x,y)$ - ...
- $\text{garden}(x,y)$ - ...
- $\text{lift}(x)$ - ...

(2)

- $\text{pets}(x)$ - ...
- $\text{central}(x)$ - ...
- $\text{acceptable}(x)$ – flat x satisfies our requirements
- $\text{offer}(x,y)$ – we pay the amount y for the flat x

(3)

Rules:

- $r1: \rightarrow \text{acceptable}(x)$
- $r2: \text{bedrooms}(x,y), y < 2 \rightarrow \text{not acceptable}(x)$
- $r3: \text{size}(x,y), y < 45 \rightarrow \text{not acceptable}(x)$
- $r4: \text{not pets}(x) \rightarrow \text{not acceptable}(x)$
- $r5: \text{floor}(x,y), y > 2, \text{not lift}(x) \rightarrow \text{not acceptable}(x)$
- $r6: \text{price}(x,y), y > 400 \rightarrow \text{not acceptable}(x)$

(4)

Calculating the offer for an apartment

r7: size(x,y), y > 45, garden(x,z), central(x)

→ offer(x, 300 + 2z + 5(y - 45))

r8: size(x,y), y > 45, garden(x,z), not(central(x))

→ offer(x, 250 + 2z + 5(y - 45))

r9: offer(x,y), price(x,z), y < z → not acceptable(x)

(5)

After finding the acceptable apartments, we need to select one according to our preferences

_ r10: cheapest(x) → rent(x)

_ r11: cheapest(x), largestgarden(x) → rent(x)

_ r12: cheapest(x), largestgarden(x), largest(x)

→ rent(x)

r12 > r11 > r10

Rule Markup in XML *-Monotonic Rules(1)*

- Terms

Used tags :<term>,<function>,<var>,<const>

f(x,a,g(b,y))

- Atomic formulas

Additional tags:<atom>,<predicate>

p(x,a,f(b,y))

-Monotonic Rules(2)

- Facts

An atomic formula enclosed by <fact> tags

- Rules

Head: an atomic formula

Body: a sequence of atomic formulas

p(x,a),q(y,b)→r(x,y)

-Monotonic Rules(3)

Queries

Are represented as bodies of rules surrounded by

<query> tags

A DTD

<!ELEMENT program((rule|fact)*)>

<!ELEMENT fact(atom)>

<!ELEMENT rule (head,body)>

<!ELEMENT head(atom)>

-Monotonic Rules(4)

<!ELEMENT body(atom*)>

<!ELEMENT atom(predicate,term*)>

<!ELEMENT term(const|var|function,term*)>

Predicates,function symbols,constants,variables are atomic types

<!ELEMENT predicate (#PCDATA)

<!ELEMENT function (#PCDATA)

<!ELEMENT var (#PCDATA)

Rule Markup in XML

-Nonmonotonic Rules

Differences between non- and monotonic rules:

Nonmonotonic rules have:

- _ No function symbols
- _ Negated atoms may occur in the head and the body of the rule
- _ Each rule has a label
- _ A program contains priority statements

Example of defensible program

r1: $p(x) \rightarrow s(x)$

r2: $q(x) \rightarrow \text{non } s(x)$

$p(a) ; q(a)$

$r1 > r2$

Representation: $\langle \text{rule id} = \text{„r1“} \rangle \dots$

-priorities : $\langle \text{stronger superior} = \text{„r1“} \text{ inferior} = \text{„r2“} \rangle$

...

A DTD(1)

```
<! ELEMENT program ((rule|fact|stronger)*)>
```

```
<! ELEMENT fact (atom|neg)>
```

```
<! ELEMENT neg (atom)>
```

```
<! ELEMENT rule|(head,body)>
```

```
<ATTLIST rule
```

```
    id ID #IMPLIED>
```

A DTD (2)

```
<! ELEMENT head (atom|neg)>
```

```
<! ELEMENT body ((atom|neg)*)>
```

Priorities:

```
<! ELEMENT stronger (EMPTY)>
```

```
<!ATTLIST stronger
```

```
    superior IDREF #REQUIRED
```

```
    inferior IDREF #REQUIRED
```


A DTD (3)

Predicates, constants and variables are atomic types

```
<! ELEMENT predicate (#PCDATA)>
```

```
<! ELEMENT var (#PCDATA)>
```

```
<! ELEMENT const (#PCDATA)>
```