Chapter 15:

Planning with relational actions

Relational Actions and Planning

- Agents reason in time.
- Agents reason about time.
- Time passes as an agent acts and reasons.
- Given a goal, it is useful for an agent to think about what it will do in the future to determine what it will do now.
- Relational representations allow to reason about actions even before the individual objects of the domain become known to the agent.

Representing Time

Time can be modeled in a number of ways:

Discrete time	Time is modeled as jumping from one time point to another.
Continuous time	Time is modeled as being dense.
Event-based time	Time steps don't have to be uniform; time steps can be between interesting events.
State space	Instead of considering time explicitly, actions can map from one state to another.

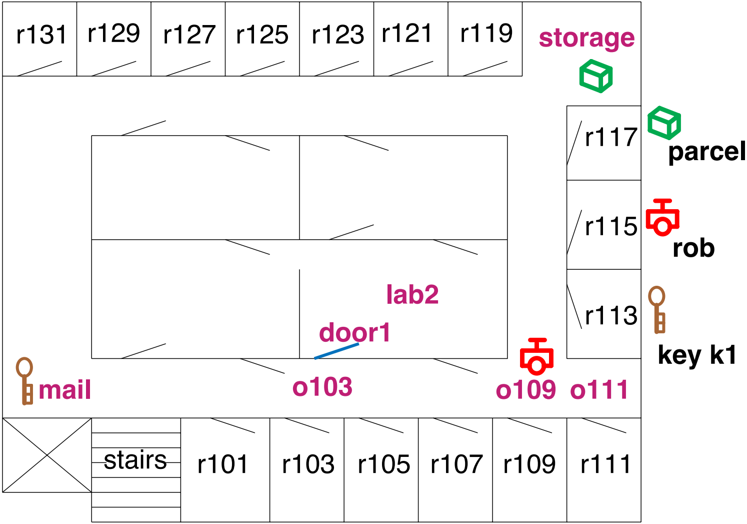
You can model time in terms of points or intervals.

Time and Relations

When modeling relations, you distinguish two basic types:

- Static relations are those relations whose value does not depend on time.
- Dynamic relations are relations whose truth values depends on time. Either
  - derived relations whose definition can be derived from other relations for each time,
  - primitive relations whose truth value can be determined by considering previous times.

The Delivery Robot World



Modeling the Delivery Robot World

- Individuals: rooms, doors, keys, parcels, and the robot.
- Actions:

  move from room to room
  pick up and put down keys and packages
  unlock doors (with the appropriate keys)
- Relations: represent

  the robot's position
  the position of packages and keys and locked doors
  what the robot is holding

Example Relations

- $at(Obj, Loc)$ is true in a situation if object *Obj* is at location *Loc* in the situation.
- $carrying(Ag, Obj)$ is true in a situation if agent *Ag* is carrying *Obj* in that situation.
- $sitting\_at(Obj, Loc)$ is true in a situation if object *Obj* is sitting on the ground (not being carried) at location *Loc* in the situation.
- $unlocked(Door)$ is true in a situation if door *Door* is unlocked in the situation.
- $autonomous(Ag)$ is true if agent *Ag* can move autonomously. This is static.

Example Relations (cont.)

- $opens(Key, Door)$ is true if key Key opens door $Door$. This is static.
- $adjacent(Pos_1, Pos_2)$ is true if position Pos_1 is adjacent to position Pos_2 so that the robot can move from Pos_1 to Pos_2 in one step.
- $between(Door, Pos_1, Pos_2)$ is true if $Door$ is between position Pos_1 and position Pos_2. If the door is unlocked, the two positions are adjacent.

Actions

- $move(Ag, From, To)$ agent Ag moves from location $From$ to adjacent location To. The agent must be sitting at location $From$.
- $pickup(Ag, Obj)$ agent Ag picks up Obj. The agent must be at the location that Obj is sitting.
- $putdown(Ag, Obj)$ the agent Ag puts down Obj. It must be holding Obj.
- $unlock(Ag, Door)$ agent Ag unlocks $Door$. It must be outside the door and carrying the key to the door.

Static Facts

$sitting\_at(rob, o109).$
$sitting\_at(parcel, storage).$
$sitting\_at(k1, mail).$

$between(door1, o103, lab2).$
$opens(k1, door1).$
$autonomous(rob).$

Derived Relations

$at(Obj, Pos) \leftarrow sitting\_at(Obj, Pos).$

$at(Obj, Pos) \leftarrow carrying(Ag, Obj) \land at(Ag, Pos).$

$adjacent(o109, o103).$

$adjacent(o103, o109).$

$\ldots$

$adjacent(lab2, o109).$

$$
\begin{aligned}
&adjacent(P_1, P_2) \leftarrow \\
&\quad between(Door, P_1, P_2) \land \\
&\quad unlocked(Door).
\end{aligned}
$$

Function Symbols

- Often we want to refer to individuals in terms of components.
- Examples: 4:55 p.m. English sentences. A classlist.
- We extend the notion of term. So that a term can be $f(t_1, \ldots, t_n)$ where f is a function symbol and the t_i are terms.
- In an interpretation and with a variable assignment, term $f(t_1, \ldots, t_n)$ denotes an individual in the domain.
- Function symbols can be recursively embedded:
  - One function symbol and one constant can refer to infinitely many individuals.
  - Function symbols can be used to describe sequential structures.

Example: Lists

- A list is an ordered sequence of elements.
- Let's use the constant *nil* to denote the empty list, and the function $cons(H, T)$ to denote the list with first element H and rest-of-list T. These are not built-in.
- The list containing *sue*, *kim* and *randy* is

$$cons(sue, cons(kim, cons(randy, nil)))$$

- $append(X, Y, Z)$ is true if list Z contains the elements of X followed by the elements of Y

$$append(nil, Z, Z).$$

$$append(cons(A, X), Y, cons(A, Z)) \leftarrow append(X, Y, Z).$$

Situation Calculus

- State-based representation where the states are denoted by terms.
- A situation is a term that denotes a state.
- There are two ways to refer to states:
  - $init$ denotes the initial state
  - $do(A, S)$ denotes the state resulting from doing action A in state S, if it is possible to do A in S.
- Time is reified by means of situations which can be entered.
  - Individuals represent points in time.
- A situation encodes how to get to the state it denotes.
  - A state may be represented by multiple situations.
  - A state may be represented by no situations if it is unreachable.
  - A situation may represent no states, if an action was not possible.

Example Situations

- $init$
- $do(move(rob, o109, o103), init)$
- $do(move(rob, o103, mail),$
  $do(move(rob, o109, o103),$
  $init)).$
- $do(pickup(rob, k1),$
  $do(move(rob, o103, mail),$
  $do(move(rob, o109, o103),$
  $init))).$

Using the Situation Terms

- Add an extra term to each dynamic predicate indicating the situation.
- Example Atoms:

$$at(rob, o109, init)$$

$$at(rob, o103, do(move(rob, o109, o103), init))$$

$$at(k1, mail, do(move(rob, o109, o103), init))$$

Axiomatizing using the Situation Calculus

- You specify what is true in the initial state using axioms with *init* as the situation parameter.
- Primitive relations are axiomatized by specifying what is true in situation $do(A, S)$ in terms of what holds in situation S.
- Derived relations are defined using clauses with a free variable in the situation argument.
- Static relations are defined without reference to the situation.

Derived Relations

$sitting\_at(rob, o109, init).$

$sitting\_at(parcel, storage, init).$

$sitting\_at(k1, mail, init).$

$adjacent(P_1, P_2, S) \leftarrow$
$\quad between(Door, P_1, P_2) \wedge$
$\quad unlocked(Door, S).$
$adjacent(lab2, o109, S).$
$\ldots$

When are actions possible?

$poss(A, S)$ is true if action A is possible in situation S.

$$
\begin{aligned}
&poss(putdown(Ag, Obj), S) \leftarrow \\
&\qquad carrying(Ag, Obj, S).
\end{aligned}
$$

$$
\begin{aligned}
&poss(move(Ag, Pos_1, Pos_2), S) \leftarrow \\
&\qquad autonomous(Ag) \land \\
&\qquad adjacent(Pos_1, Pos_2, S) \land \\
&\qquad sitting\_at(Ag, Pos_1, S).
\end{aligned}
$$

Axiomatizing Primitive Relations

Example: Unlocking the door makes the door unlocked:

$$unlocked(Door, do(unlock(Ag, Door), S)) \leftarrow$$
$$poss(unlock(Ag, Door), S).$$

Frame Axiom: No actions lock the door:

$$unlocked(Door, do(A, S)) \leftarrow$$
$$unlocked(Door, S) \wedge$$
$$poss(A, S).$$

Example: axiomatizing *carried*

Picking up an object causes it to be carried:

$$\begin{aligned}&carrying(Ag, Obj, do(pickup(Ag, Obj), S)) \leftarrow\\ &\quad poss(pickup(Ag, Obj), S).\end{aligned}$$

Frame Axiom: The object is being carried if it was being carried before unless the action was to put down the object:

$$\begin{aligned}&carrying(Ag, Obj, do(A, S)) \leftarrow\\ &\quad carrying(Ag, Obj, S) \wedge\\ &\quad poss(A, S) \wedge\\ &\quad A \neq putdown(Ag, Obj).\end{aligned}$$

Example: *sitting_at*

An object is sitting at a location if:

- it moved to that location:

$$sitting\_at(Obj, Pos, do(move(Obj, Pos_0, Pos), S)) \leftarrow$$
$$poss(move(Obj, Pos_0, Pos), S).$$

- it was put down at that location:

$$sitting\_at(Obj, Pos, do(putdown(Ag, Obj), S)) \leftarrow$$
$$poss(putdown(Ag, Obj), S) \wedge$$
$$at(Ag, Pos, S).$$

- it was at that location before and didn't move and wasn't picked up.

More General Frame Axioms

The only actions that undo *sitting_at* for object *Obj* is when *Obj* moves somewhere or when someone is picking up *Obj*.

$$\begin{aligned}
&sitting\_at(Obj, Pos, do(A, S)) \leftarrow \\
&\quad poss(A, S) \wedge \\
&\quad sitting\_at(Obj, Pos, S) \wedge \\
&\quad \forall Pos_1 \;\; A \neq move(Obj, Pos, Pos_1) \wedge \\
&\quad \forall Ag \;\; A \neq pickup(Ag, Obj).
\end{aligned}$$

More General Frame Axioms

$$\forall Ag \quad A \neq pickup(Ag, Obj)$$

is equivalent to:

$$\sim \exists Ag \ A = pickup(Ag, Obj)$$

which can be implemented as

$$\begin{aligned} & sitting\_at(Obj, Pos, do(A, S)) \leftarrow \\ & \qquad \cdots \wedge \ \cdots \wedge \ \cdots \wedge \\ & \qquad \sim is\_pickup\_action(A, Obj). \end{aligned}$$

with the clause:

$$\begin{aligned} & is\_pickup\_action(A, Obj) \leftarrow \\ & \qquad A = pickup(Ag, Obj). \end{aligned}$$

which is equivalent to:

$$is\_pickup\_action(pickup(Ag, Obj), Obj).$$

Planning

Given

- an initial world description
- a description of available actions
- a goal

a **plan** is a sequence of actions that will achieve the goal.

Example Planning

If you want a plan to achieve Rob holding the key $k1$ and being at $o103$, the query

$$?\ carrying(rob, k1, S) \wedge at(rob, o103, S).$$

has an answer

$$\begin{aligned} S = do(&move(rob, mail, o103),\\ &do(pickup(rob, k1),\\ &\quad do(move(rob, o103, mail),\\ &\quad\quad do(move(rob, o109, o103), init)))). \end{aligned}$$

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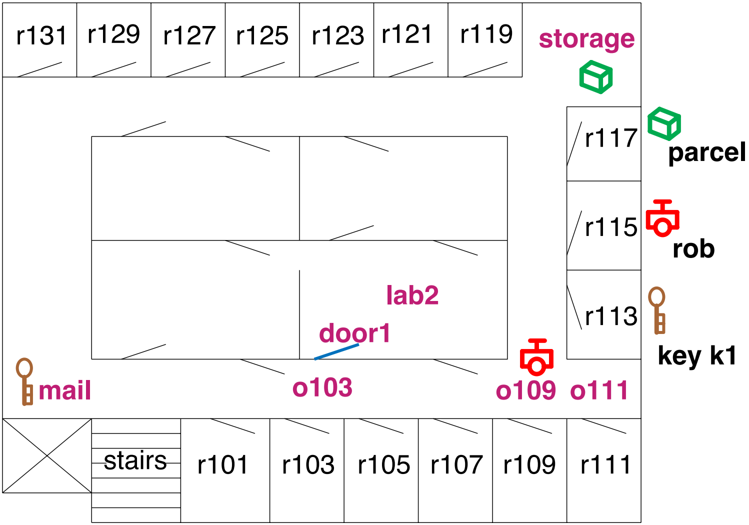

Planning as Resolution

- Idea: backward chain on the situation calculus rules.
- A complete search strategy (e.g., A^* or iterative deepening) is guaranteed to find a solution.
- When there is a solution to the query with situation $S = do(A, S_1)$, action A is the last action in the plan.
- You can virtually always use a frame axiom so that the search space is largely unconstrained by the goal. Search space is enormous.

Goal-directed searching

- Given a goal, you would like to consider only those actions that actually achieve it.
- Example:

  $$? \; carrying(rob, parcel, S) \wedge in(rob, lab2, S).$$

  the last action needed is irrelevant to the left subgoal.
- So we need to combine the planning algorithms with the relational representations.