

# Database and Information Systems

11. Deductive Databases
12. Data Warehouses and OLAP
13. **Data Mining**
14. Index Structures for Similarity Queries
15. Semi-Structured Data
16. Document Retrieval
17. Web Mining
18. Content Extraction
19. Multimedia Data

# Data Mining

- Data-Mining: The Task
- Data-Mining as a Process
- Data Preprocessing
- Data Mining Tasks

# Data Mining Tasks

- Classification
- Prediction
- Clustering
- Dependency Modelling
- Summarization
- Change and Deviation Detection
- Visualization

# Data Mining Tasks

- Classification
- Prediction
- Clustering
- Dependency Modelling
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# Classification

- well understood
  - decision theory
  - many heuristic solutions
- applications in
  - customer relationship management: tailored marketing
  - banking: credit authorization
  - document management: e-mail routing

# Classification

- given
  - a data base  $D = \{t_1, t_2, \dots, t_n\}$
  - of tuples  $t_i = \vec{x}$  and
  - a set of classes  $C = \{c_1, c_2, \dots, c_m\}$ ,
- find a mapping  $f : D \rightarrow C$ 
  - such that  $f$  partitions  $D$ .

# Classification

- classes are predefined:  
supervised learning, learning with a teacher
- notation
  - $c_t(t_i)$ : class assignment in the training data
  - $c(t_i)$ : class assignment by the classifier
- usually:  $|D| \gg |C|$
- class is a set of tuples:  $c_j = \{t_i | f(t_i) = c_j\}$
- no tuple belongs to several classes

# Classification

- Nearest-Neighbor Classifier
- Threshold-based Classifiers
- Decision Trees
- Random Forests
- Neural Networks
- Stochastic Classification
- Evaluation



# Classification

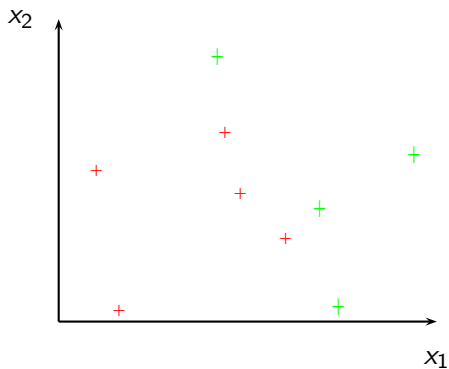
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# Nearest-Neighbor Classifier

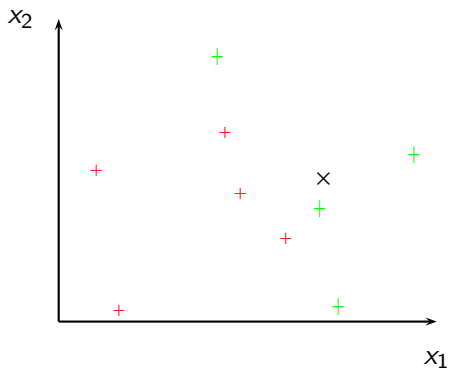
- direct approach: training data  $T = \{(\vec{x}_i, k)\}$  are
  - directly stored in the classifier and
  - used for classification
- nearest neighbor

$$c(\vec{x}) = \arg_{c_k}(\vec{x}_j, k), \quad j = \arg \min_i d(\vec{x}, \vec{x}_i)$$

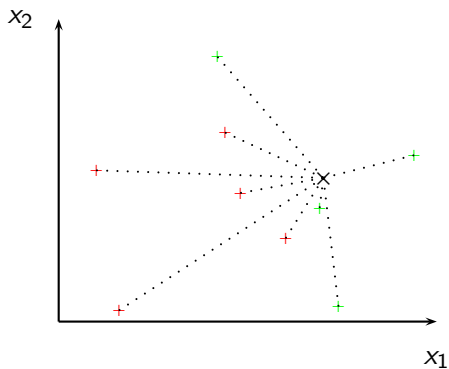
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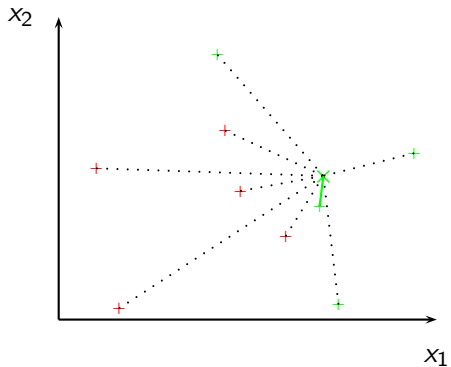
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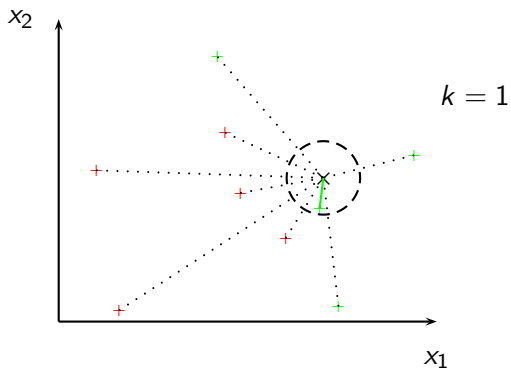
# Nearest-Neighbor Classifier

- $k$ -nearest neighbors
  - determine the set  $N$  of the  $k$  nearest neighbors of  $\vec{x}$  in  $T$
  - choose the class with the maximum number of data points in  $N$

$$c(\vec{x}) = \arg \max_{c_k} |\{c_k | c_k \in N\}|$$

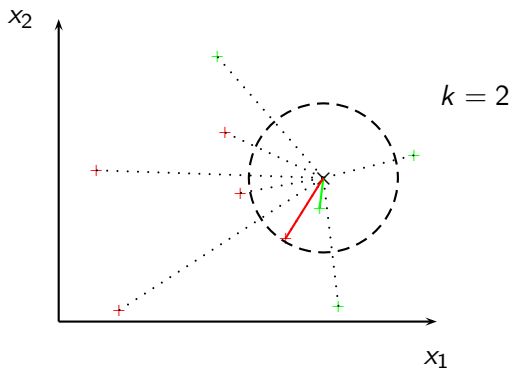
- more robust against singular data points
- but more expensive

# Nearest-Neighbor Classifier

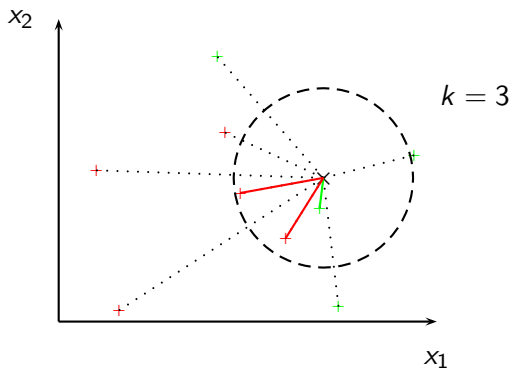




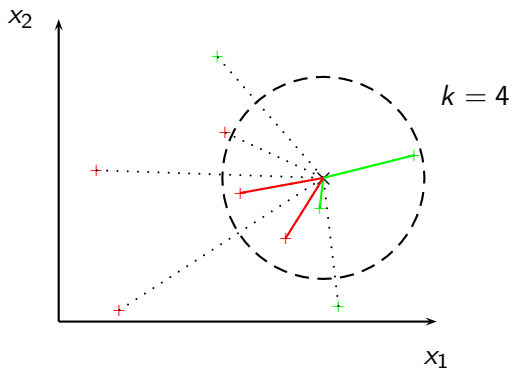
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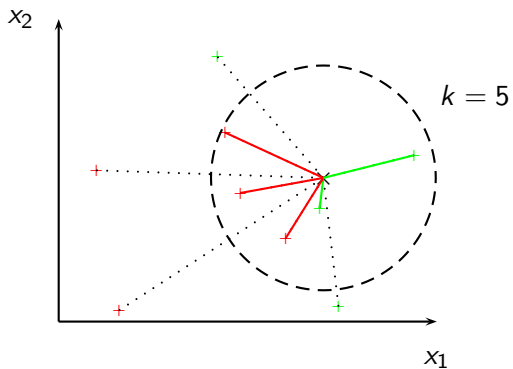
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  - no generalization of the available training data
- generalizing models required
  - use class representatives as data points
  - e.g. mean of class or class-dependent clusters

# Classification

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- **Threshold-Based Classifiers**
- Decision Trees
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# Threshold-Based Classifiers

- simple generalizing model



# Threshold-Based Classifiers

- simple generalizing model
- a threshold divides the data space into two subspaces

$$c(\vec{x}_i) = \begin{cases} 1 & x_j > \theta_j \\ 2 & \textit{else} \end{cases}$$

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$$c(\vec{x}_i) = \begin{cases} 1 & x_j > \theta_j \\ 2 & \textit{else} \end{cases}$$

- analogue separation criteria for non-numeric data

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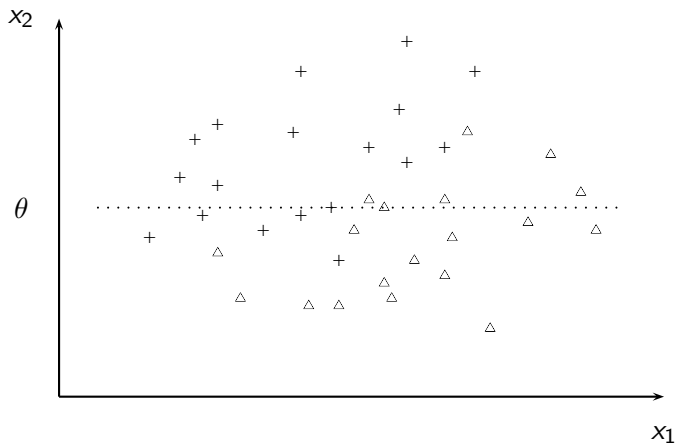
$$\theta = \arg \min_{\theta} |\{t_i | c(t_i) \neq c_t(t_i)\}|$$

- for numeric data approximated by minimizing the distance of misclassified samples to the threshold

$$\theta = \arg \min_{\theta} \sum_{t_i, c(t_i) \neq c_t(t_i)} |x_j - \theta_k|$$

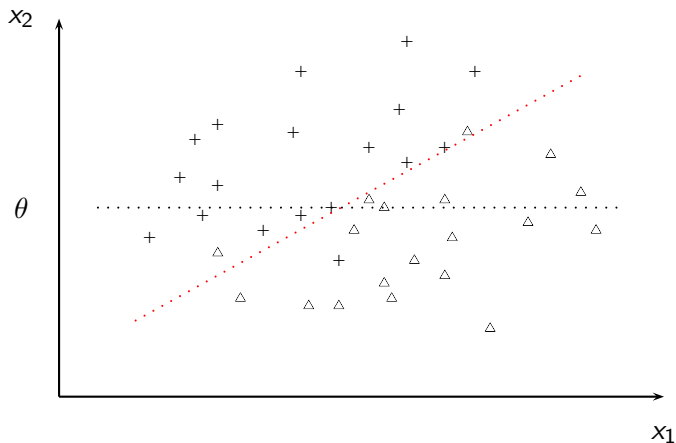
# Threshold-Based Classifiers

- insufficient to separate more difficult distributions



# Threshold-Based Classifiers

- better class separation



# Threshold-Based Classifiers

- algorithm for finding an optimal threshold
  1. sort the values  $[v_1, \dots, v_m]$
  2. extract  $m - 1$  potential thresholds by either
    - computing the mean of all neighboring values or
    - choosing the smaller one of two neighboring values
  3. evaluate all potential thresholds and select the one with the maximum gain

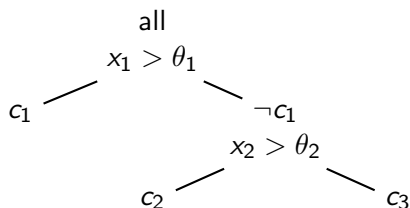


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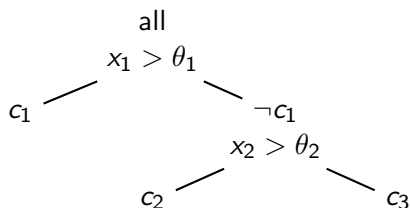
# Decision trees

- extension of threshold-based classifiers to multiple classes:
  - multi-branch splits
  - decomposition into a sequence of sub-decisions



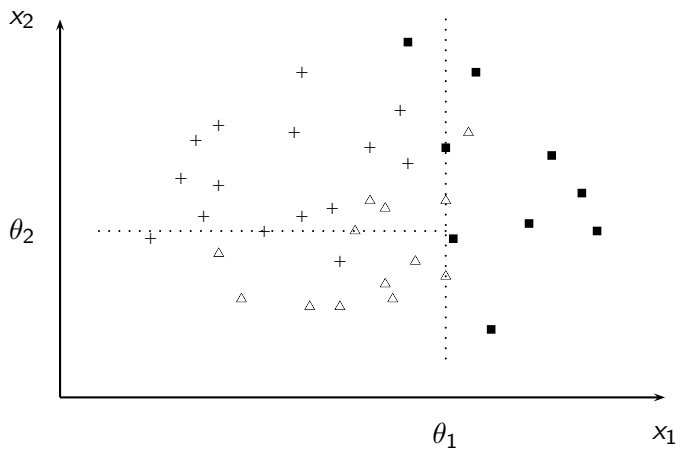
# Decision trees

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- finding the optimal decision tree is NP complete  
→ deterministic (non-backtracking), greedy algorithms

# Decision trees



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- entropy of a test set  $T$  partitioned into  $n$  subsets by an attribute test  $X$  with  $n$  possible outcomes

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- information gain of the attribute test

$$G(X) = E(T) - E_X(T)$$



# Decision trees

- C4.5: extension of ID3 to numerical data
  - split along a dimension so that the resulting subsets have lowest class entropy  
i.e. contain data points of as few classes as possible

# Decision trees

- C4.5: extension of ID3 to numerical data
  - split along a dimension so that the resulting subsets have lowest class entropy
    - i.e. contain data points of as few classes as possible
- problem of overfitting
  - splitting until no data point is misclassified usually means to adapt the classifier too much to the training data
    - "learning off by heart"
    - degrading performance on held out test data
  - cut-off criterion required, or post-pruning

# Decision trees

- decision rules can be extracted from a decision tree
  - IF part: combine all tests on the path from the root node to the leaf node
  - THEN part: the final classification

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- given:  $N$  training instances with  $M$  variables
- $T$  random samples (with replacement) with a sample size  $S \approx 0.66N$ ; the remaining instances are used for estimating the prediction error (out-of-bag evaluation)
- for each node of the tree  $m \ll M$  variables are selected randomly to take the decision at that node
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- no pruning is applied
  
- classification: (weighted) majority vote among the results of the individual trees

# Random Forests

- highly accurate classifiers without much prior knowledge
- fast training
- highly parallel classification
- can deal with very large problems (training data, number of variables, number of classes)
- estimates the importance of the different variables
- build-in evaluation; no separate test data required



# Random Forests

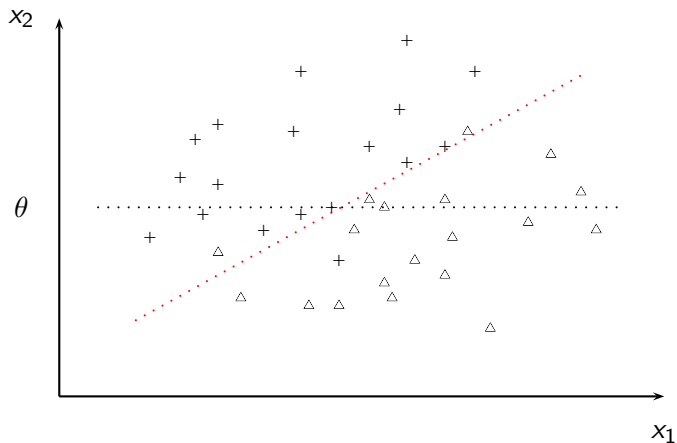
- prone to overfitting, in particular for noisy data sets
- does not handle large amounts of irrelevant features
- sensitive to correlations between trials: "small heterogeneous is better than large homogeneous" (GASHLER ET AL. 2008)
  - growing too many trees might reduce the accuracy

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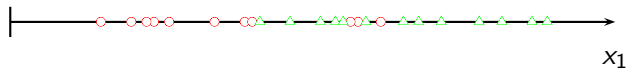
# Neural Networks

- sometimes linear functions can be used to separate two classes



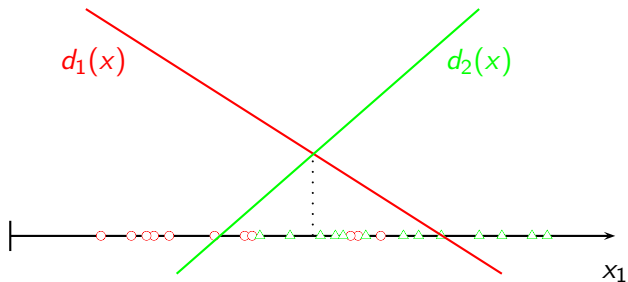
# Neural Networks

- classes are represented by means of linear discrimination functions  $d_k(\vec{x})$



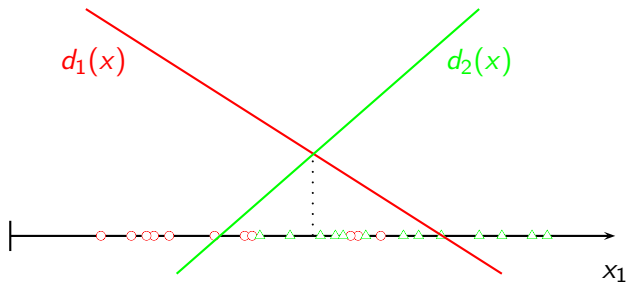
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→ linear discriminance analysis (LDA)

# Neural Networks

- class decision is reduced to a maximum detection

$$c(\vec{x}) = \arg \max_{c_k} d_k(\vec{x})$$

# Neural Networks

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$$c(\vec{x}) = \arg \max_{c_k} d_k(\vec{x})$$

- discriminating functions are (in the simplest case) linear combinations of the components of a data point

$$d_k(\vec{x}) = w_0 + \sum w_i x_i$$



# Neural Networks

- corresponds to first part of a perceptron

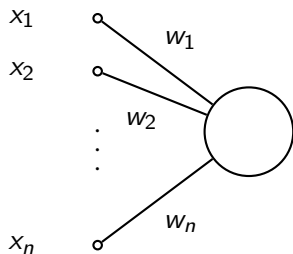
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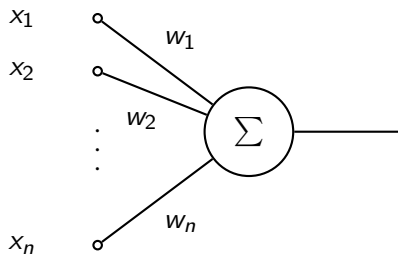
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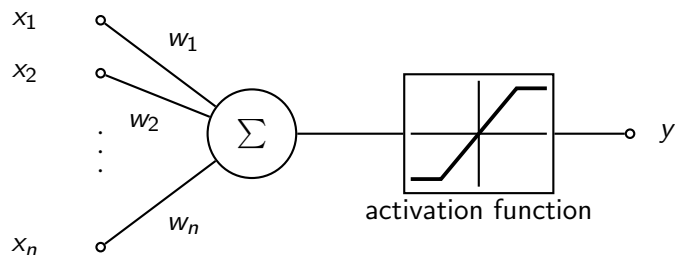
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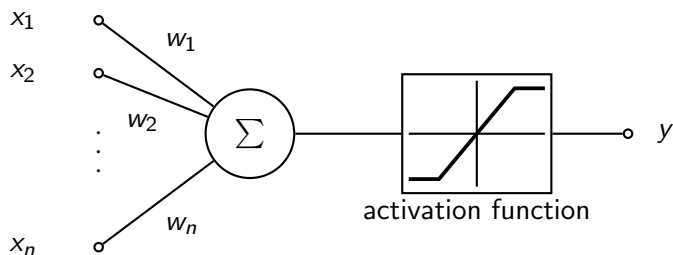
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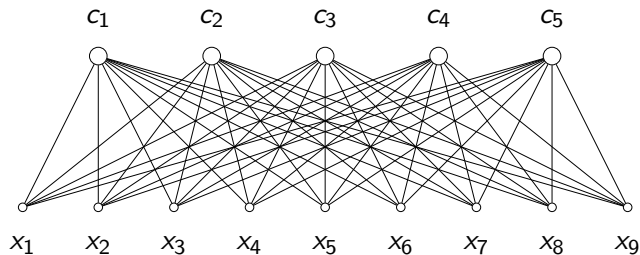
- single perceptron: classification only for
  - two class problems and
  - linear separable classes

# Neural Networks

- extension to multiple classes: single-layer networks

# Neural Networks

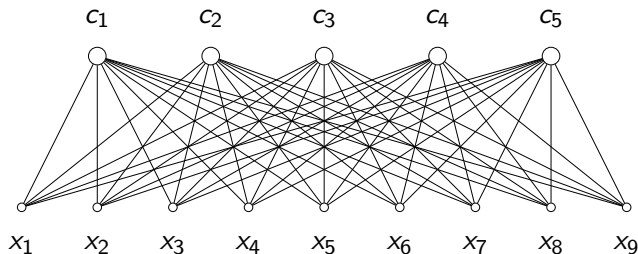
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# Neural Networks

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- class decision: maximum detection ("the winner takes all")

# Neural Networks

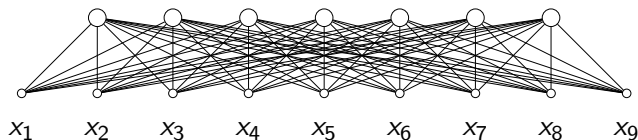
- multiple perceptrons simulate a piecewise-linear discrimination function

# Neural Networks

- multiple perceptrons simulate a piecewise-linear discrimination function
- single layer networks only for simple problems  
→ usually multiple-layer networks required

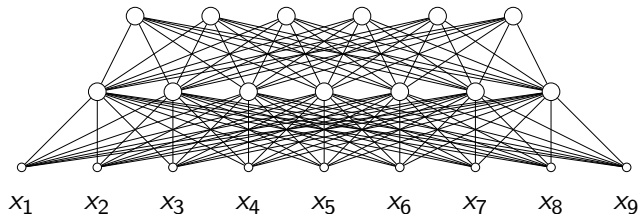
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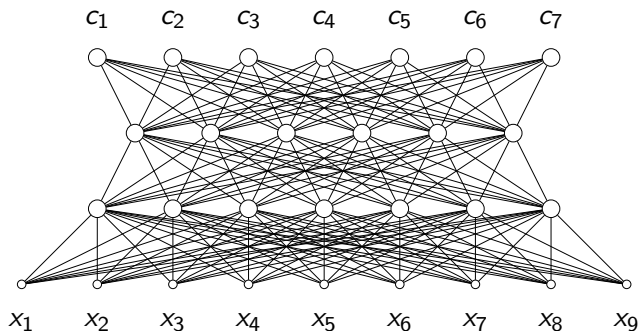
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# Neural Networks

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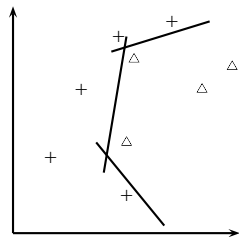
# Neural Networks

- optimal architecture has to be determined experimentally
- only few heuristic criteria available



# Neural Networks

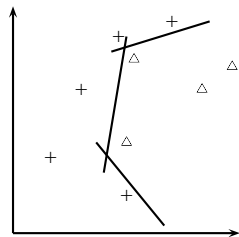
- How many layers are necessary?



single-layer network

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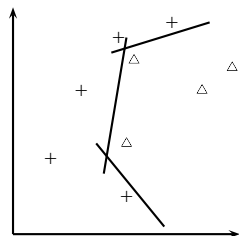


single-layer network

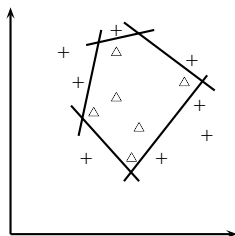
no islands

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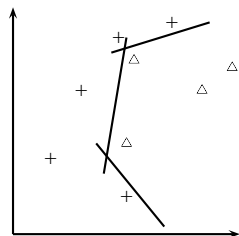


two-layer network

no islands

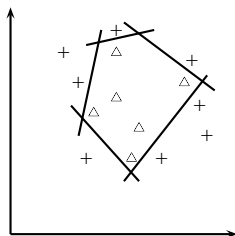
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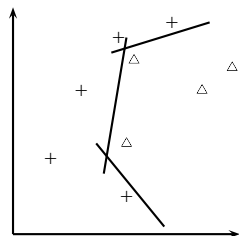


two-layer network

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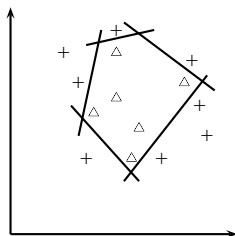
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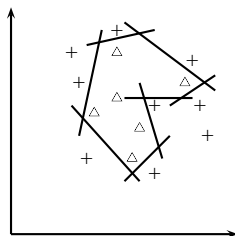
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two-layer network

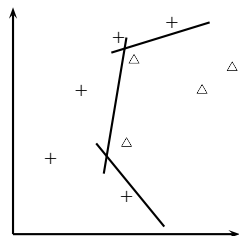
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three-layer network

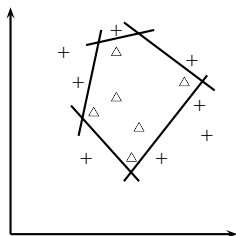
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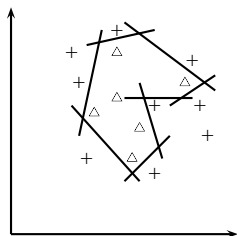
single-layer network

no islands



two-layer network

only convex islands



three-layer network

also concave islands

# Neural Networks

- How many nodes per layer are required?
  - the more nodes, the smoother the class separation
  - the more nodes, the more training data and training cycles are required

# Neural Networks

- training of neuronal networks



# Neural Networks

- training of neuronal networks
- error driven learning
  - assume an initial (random) assignment of synaptic weights  $w_{ij}$
  - determine the error of the output value of a node  $i$ :  
$$e_i = \frac{(y_i - d_i)^2}{2}$$
  - change the weights  $w_{ij}$  according to a learning rule
  - continue with the next training sample

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- backpropagation of the error signal from the output layer to the input layer

# Neural Networks

- examples of learning rules

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  - Hebb rule

$$\Delta w_{ij} = \eta x_{ij} y_j$$

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$$\Delta w_{ij} = \eta x_{ij} (d_j - y_j)$$

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- delta rule

$$\Delta w_{ij} = \eta x_{ij} (d_j - y_j)$$

- learning rules with momentum

$$\Delta w_{ij}(n) = \eta x_{ij} (d_j - y_j) + \alpha \Delta w_{ij}(n-1)$$

# Neural Networks

- gradient descent search: all weights are changed until no significant change of the global error measure can be observed
  - high number of training iterations is required
  - local search: optimum is not guaranteed
  - not even convergence of the algorithm is guaranteed

# Neural Networks

- problems



# Neural Networks

- problems
  - many parameters have to be determined empirically
    - number of layers, number of nodes per layer
    - learning rate, momentum
    - initialization
    - termination criterion

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- problems
  - many parameters have to be determined empirically
    - number of layers, number of nodes per layer
    - learning rate, momentum
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    - termination criterion
  - overfitting may occur
    - stop training early enough
    - choose the most simple architecture possible

# Neural Networks

- decision rules can be extracted from a neural network
  - cluster the node activations
  - generate rules from high synaptic links
  - combine the rules across layers

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# Stochastic Classification

- Bayesian inference
  - given: a prior data distribution
  - observe data
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- Bayesian inference
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- Bayes' theorem

$$p(c_k|\vec{x}) = \frac{p(\vec{x}|c_k) \cdot p(c_k)}{p(\vec{x})}$$

- $p(\vec{x})$  does not influence a class decision
- $p(\vec{x}|c_k)$  and  $p(c_k)$  have to be estimated using the available training data

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- training method: maximum likelihood (ML) estimation

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- Bayes classifier has optimal error rates
- but: in practice worse because of the independence assumption

# Stochastic Classification

- problems with non-trivial input output dependencies
  - strongly correlated variables
  - time series analysis
- Bayesian networks
- emission probabilities are conditioned on the state of the model
- state of the model is not directly observable → hidden variable
- simplifying assumption: state probabilities depend only on the preceding state
- ML training requires direct counting of observations
- alternative: expectation maximization (EM)
  - start with an initial probability estimation
  - modify the current probabilities as to better fit the training data
- resulting probabilities are only approximations

## Comparison of classifiers

	Nearest Neighbor	Decision Trees	(Linear) Discriminance	Stochastic Models
model	sample set	ranking of tests	class boundaries	probabilistic generation
generalization	no	forced	yes	yes
robust against incons. data outliers	no only k-NN	no low	yes yes	yes yes
perspicuity	high	high	low	low
scalability	very low	low	good	very good
additional assumptions	metrics	no	architecture learning rule	(architecture) distribution

# Classification

- Nearest-Neighbor Classifier
- Threshold-Based Classifiers
- Decision Trees
- Random Forests
- Neural Networks
- Stochastic Classification
- **Evaluation**



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- using data for training and testing: resampling

# Resampling Methods

- held out data
  - 30% ... 50% of the data are reserved for testing
  - training and test data are independent
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→ repeat the measurement with different partitionings and average
- leave one out
  - use  $n - 1$  samples for training and evaluate on the  $n$ -th one
  - repeat with all  $n$  samples
  - extremely expensive

# Resampling Methods

- n-fold cross validation
  - combines hold-out and leave-one-out
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- bootstrapping
  - generate artificial training data by replacing data items
  - obtain bootstrap estimations of the error rates on these data sets
  - useful if few data are available



# Quality Measures

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- relative improvement/degradation

$$\Delta_{rel}a = \frac{a_n - a_{n-1}}{a_n}$$

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## Quality Measures

- special case: 2 classes (true/false)  $\rightarrow$  2 error cases
- false positives/acceptance: false acceptance rate (sensitivity)

$$FAR = \frac{|\{x|c(x) = \text{true} \neq c_t(x)\}|}{|\{x|c(x) = \text{true}\}|}$$

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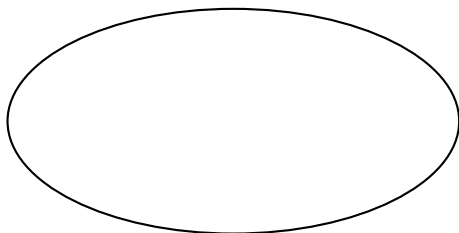
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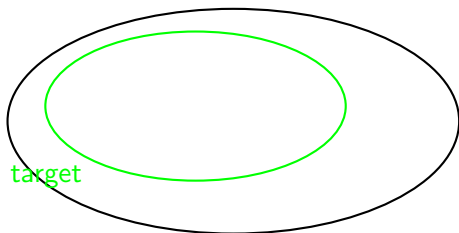
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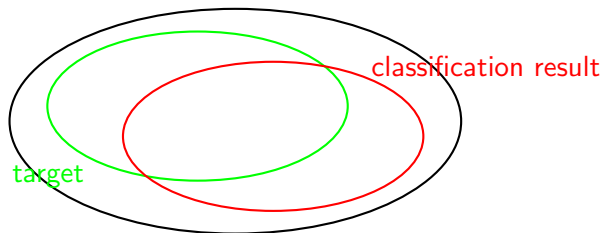
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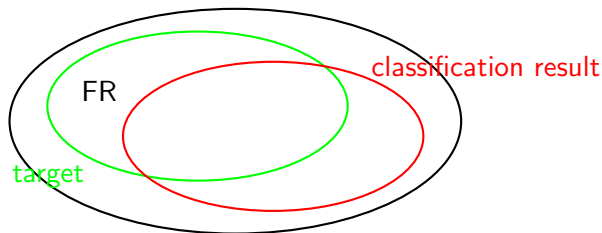
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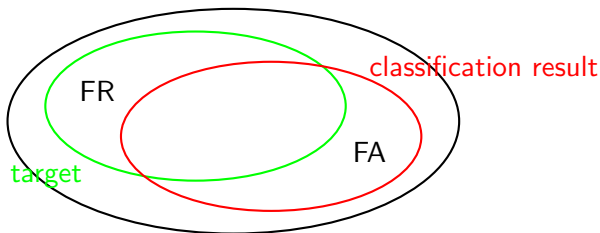
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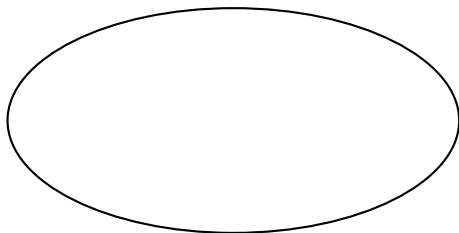
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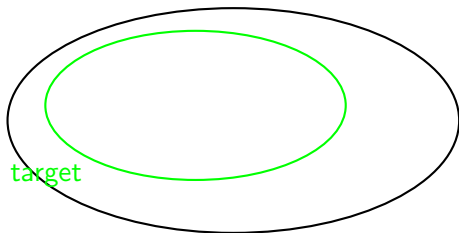
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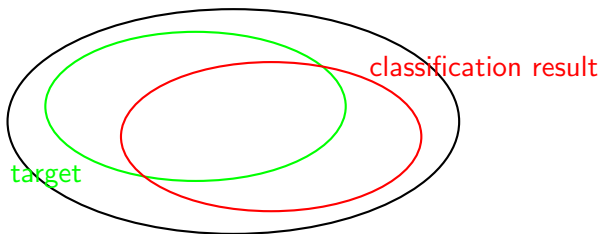
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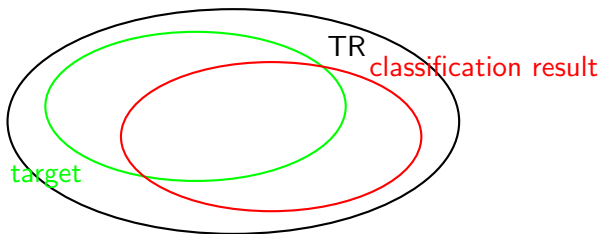
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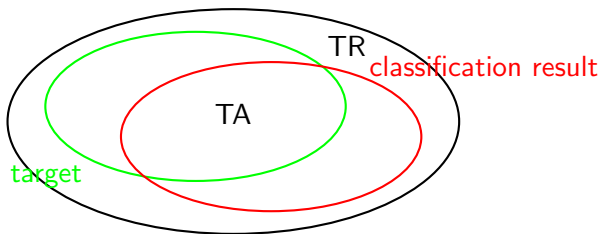
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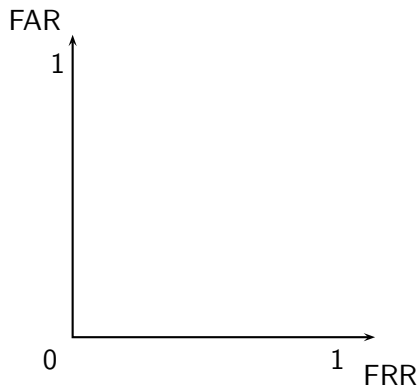
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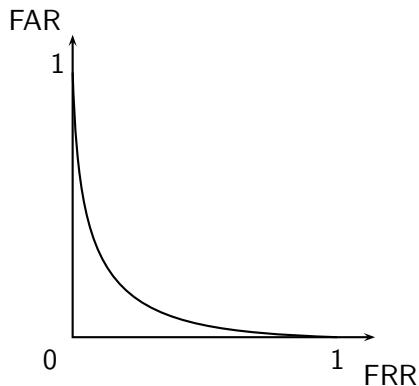
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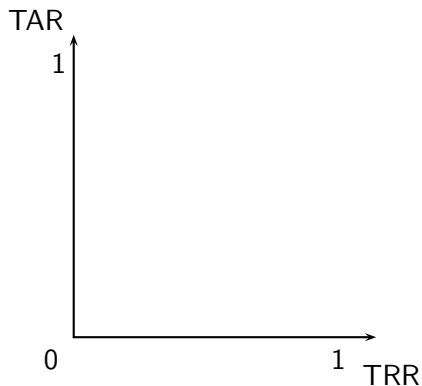
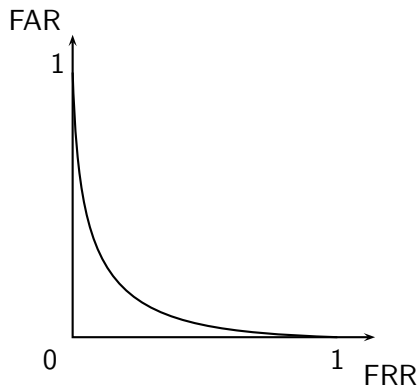
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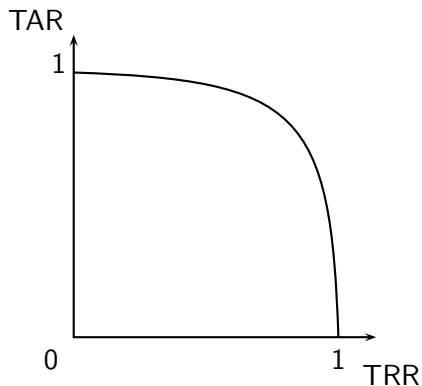
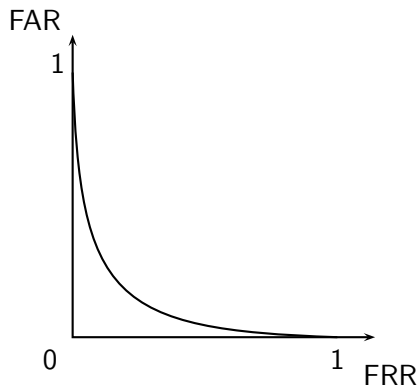
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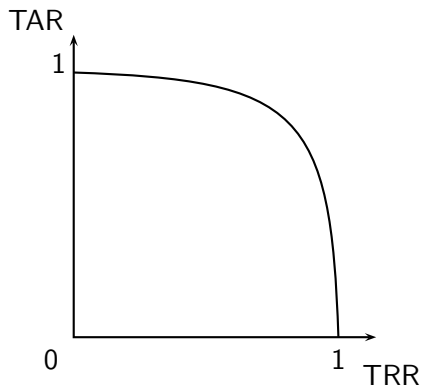
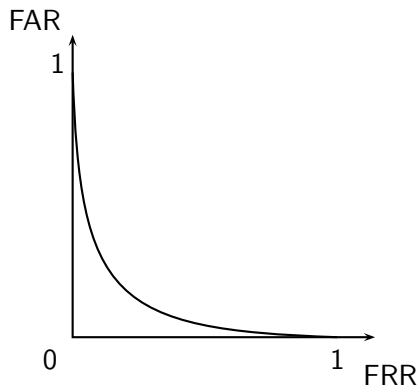
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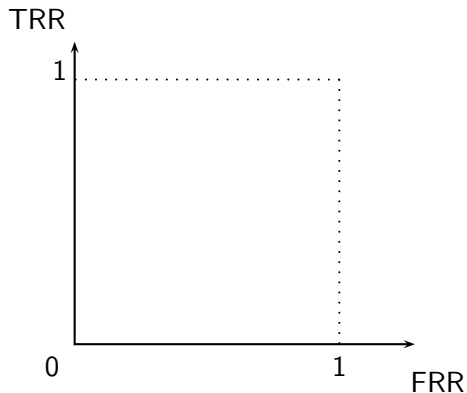


- trivial classifier: upper threshold for the error rate

$$e_{max} = \min(p(\text{true}), p(\text{false}))$$

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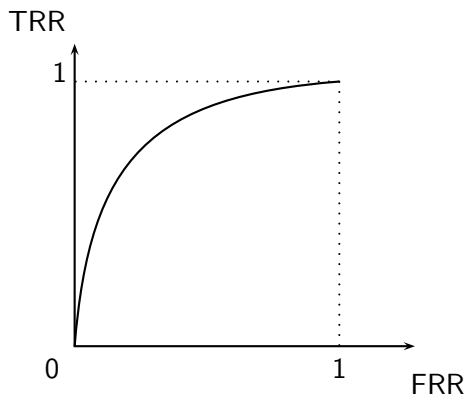
- receiver operating characteristic (ROC): TRR vs. FRR





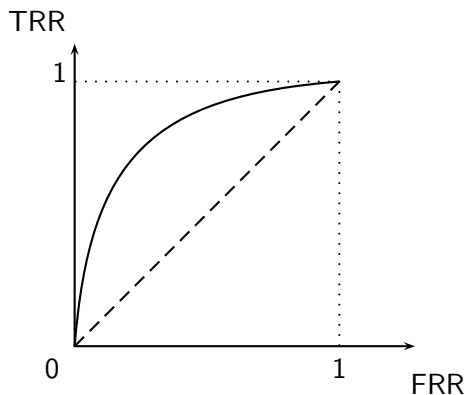
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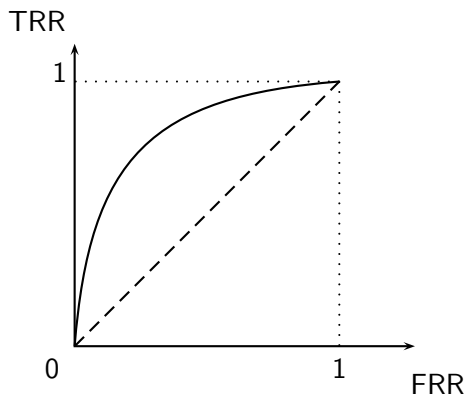
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- quality: area under the ROC-curve

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- description of the error type distribution as a confusion matrix
- biased error consequences: weighted error measures
  - error types  $e_{ij}$  are associated with costs  $c_{ij}$

$$e_w = \frac{\sum_{i=1}^m \sum_{j=1}^m e_{ij} \cdot c_{ij}}{|S|}$$